Approximating Values of Generalized-Reachability Stochastic Games

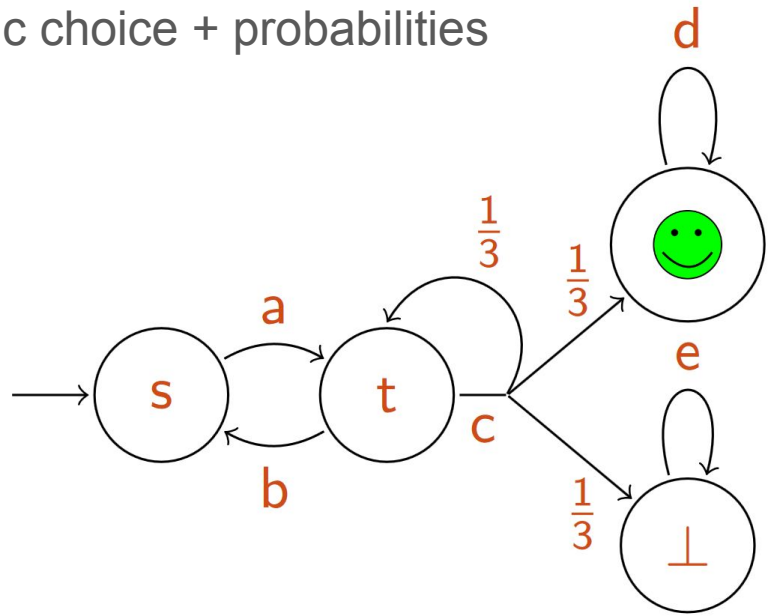
Pranav Ashok[1], Krishnendu Chatterjee[2], Jan Kretínský[1], Maximilian Weininger[1], Tobias Winkler[3]

[1] Technical University of Munich, [2] IST Austria, [3] RWTH Aachen

Appeared at LICS 2020

Recap

- Markov decision process: Non-deterministic choice + probabilities



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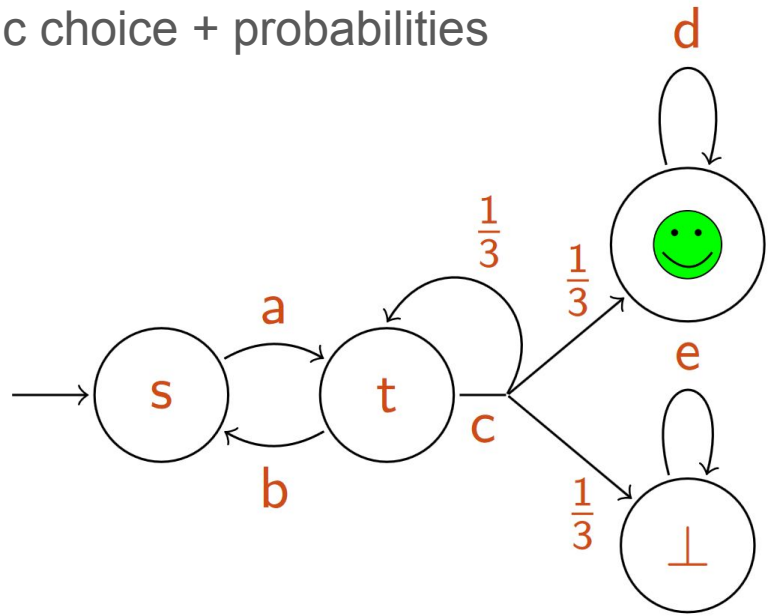

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Iteration	x_i(s)	x_i(t)
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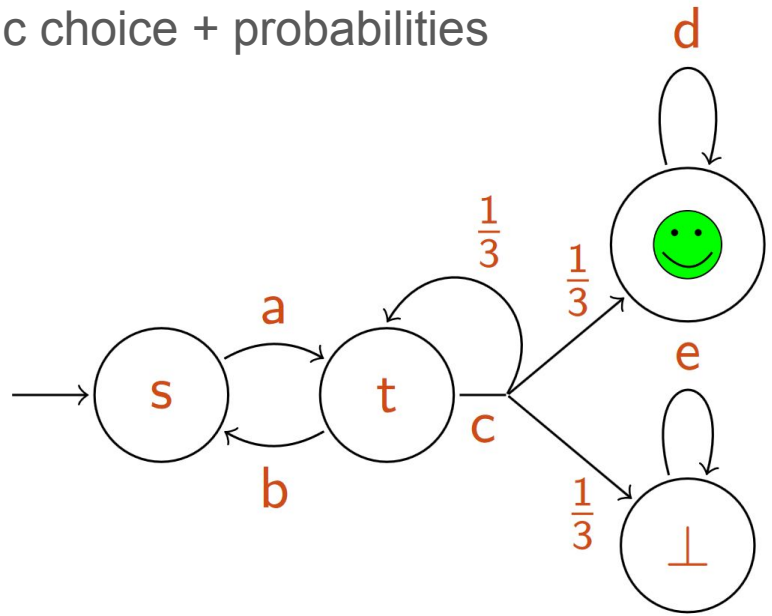
1		
2		
…		



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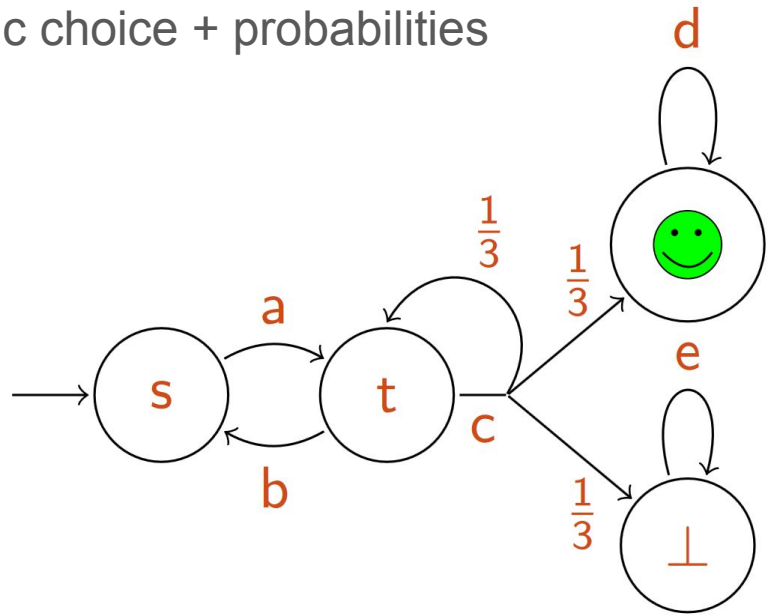

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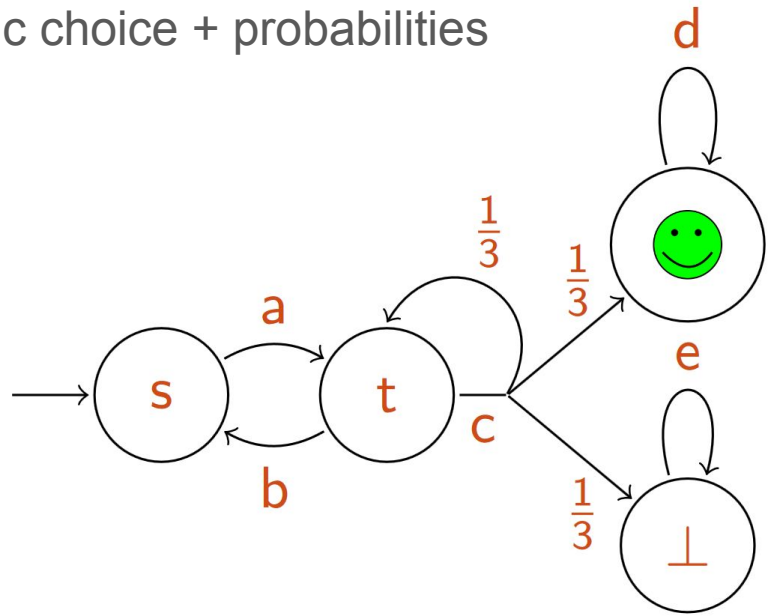

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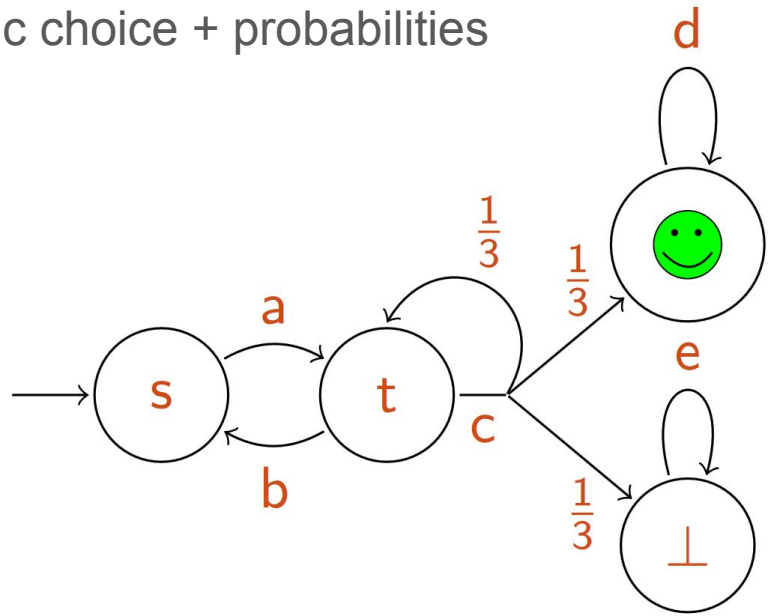

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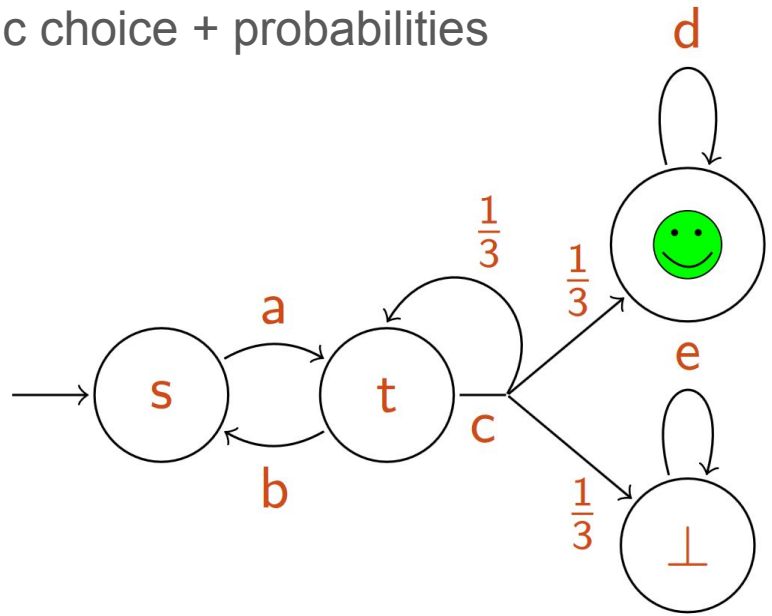

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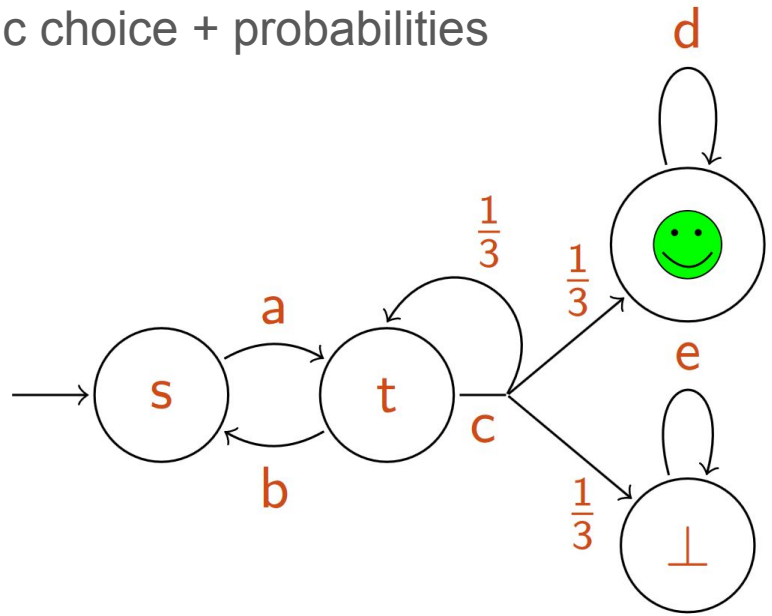

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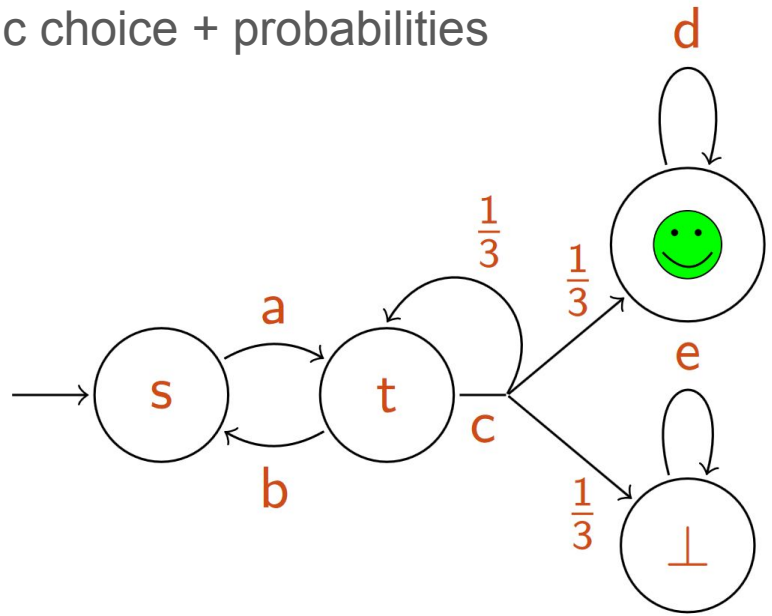
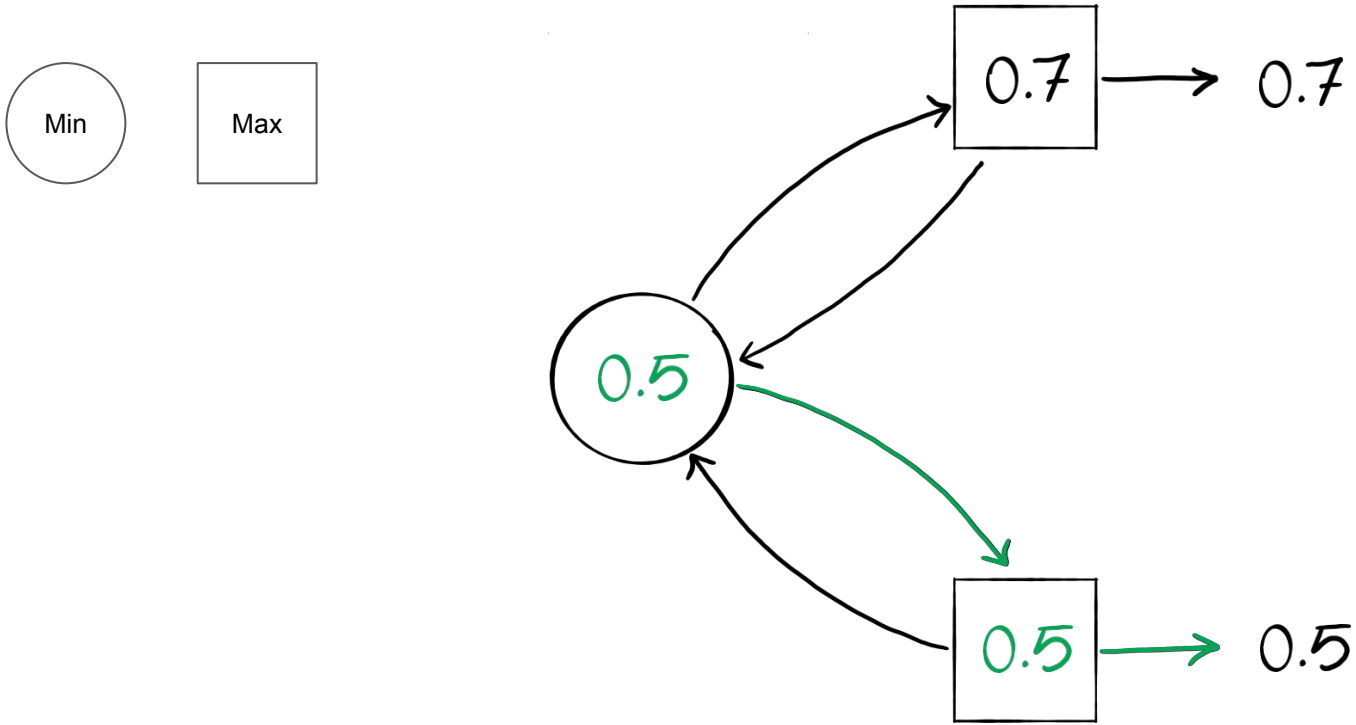
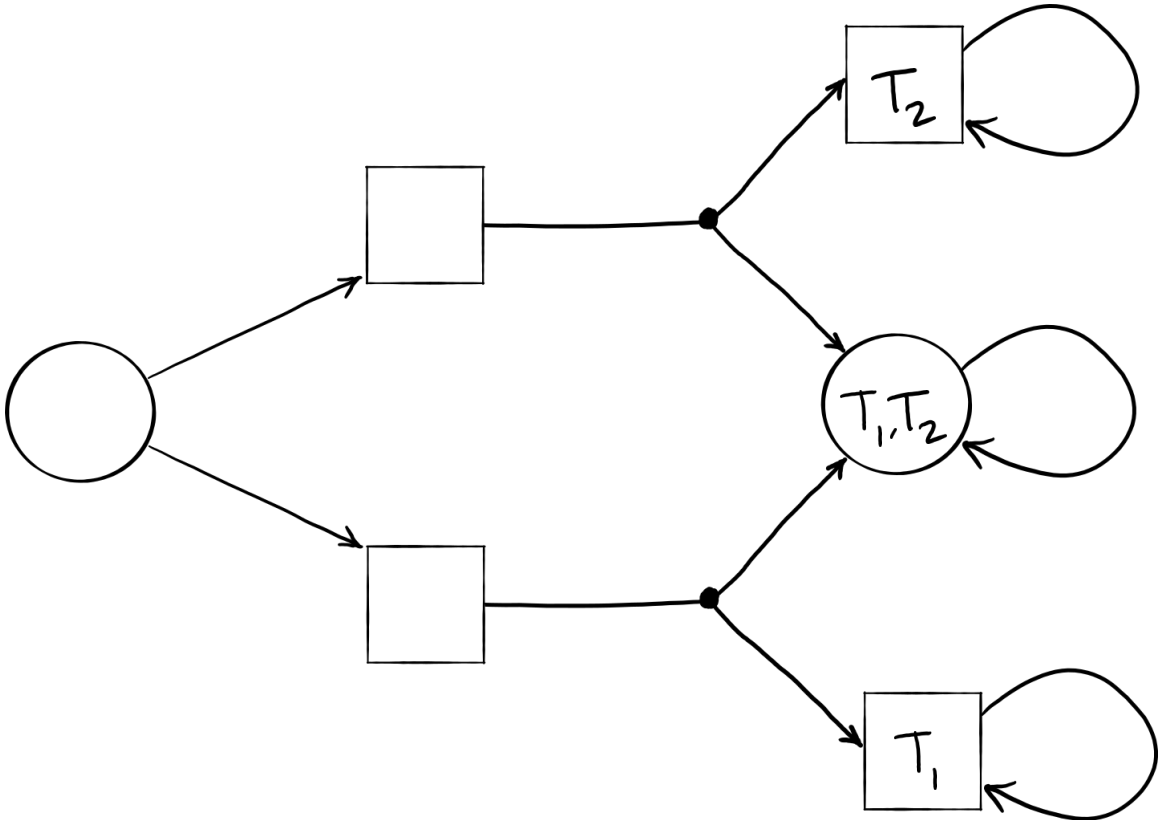


We add two things: Another player and multiple objectives

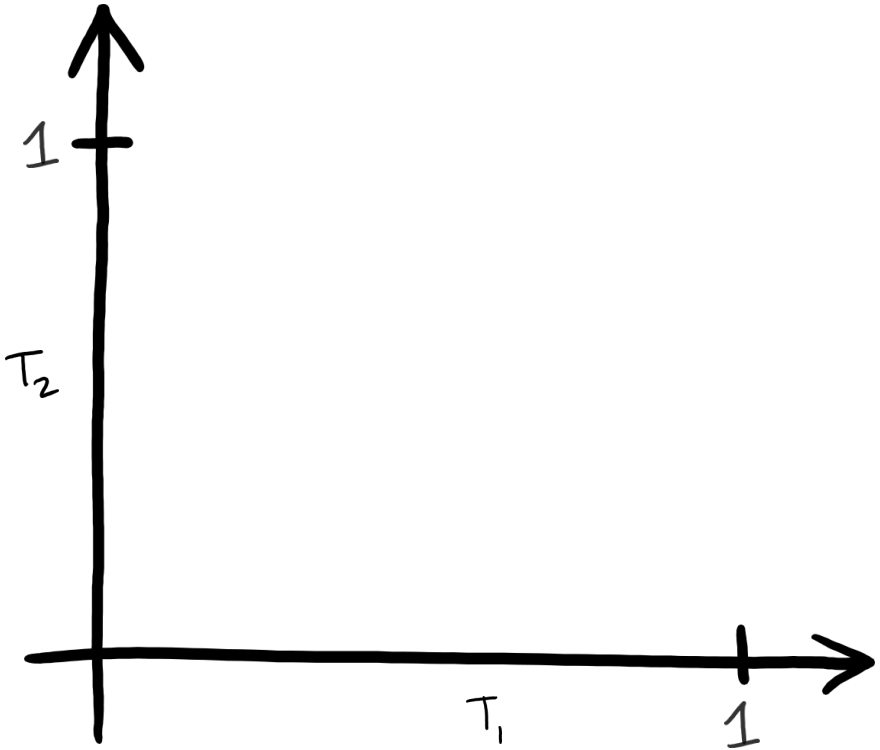
Adding a second player: Stochastic Games



Having multiple target sets



Problem statement



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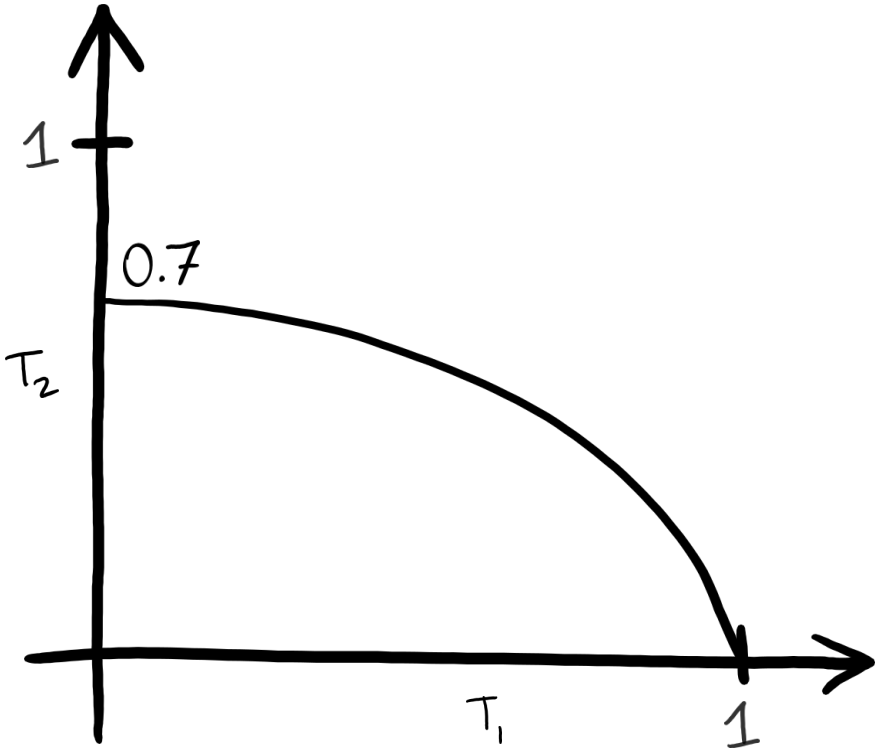

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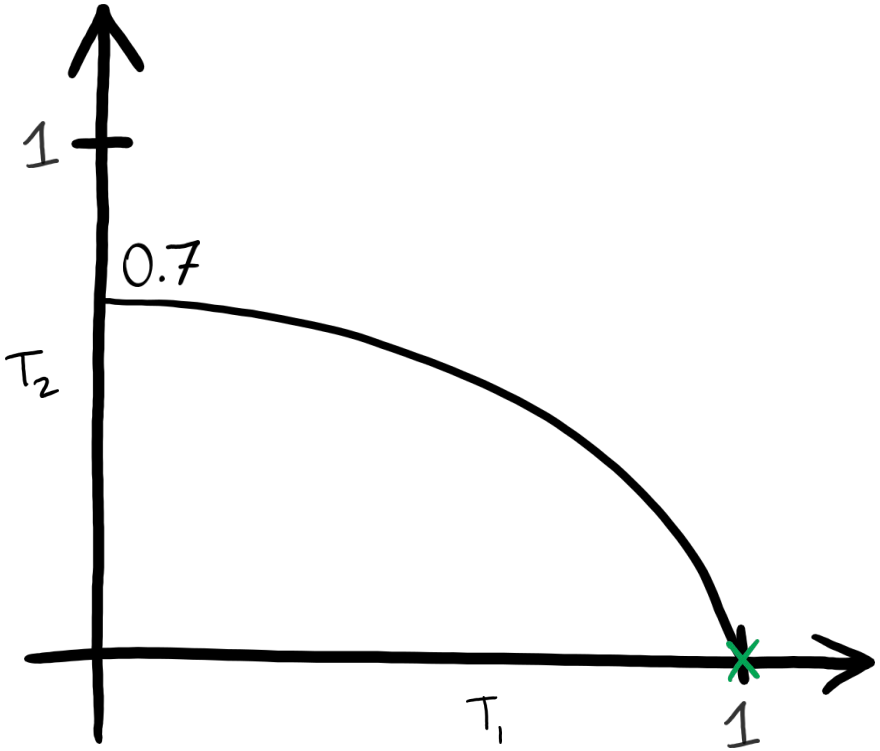

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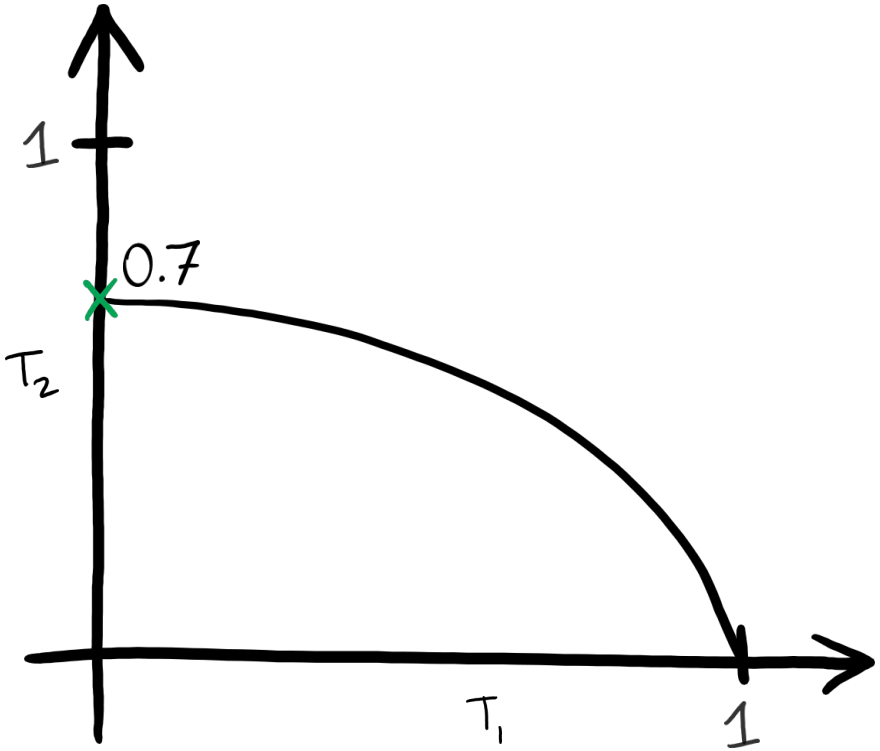

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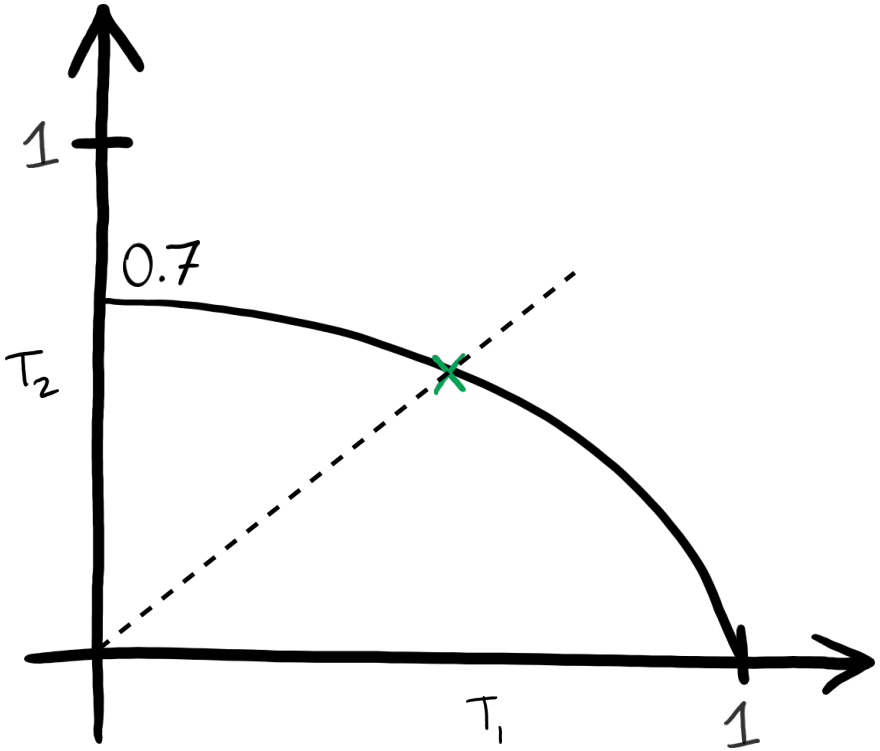

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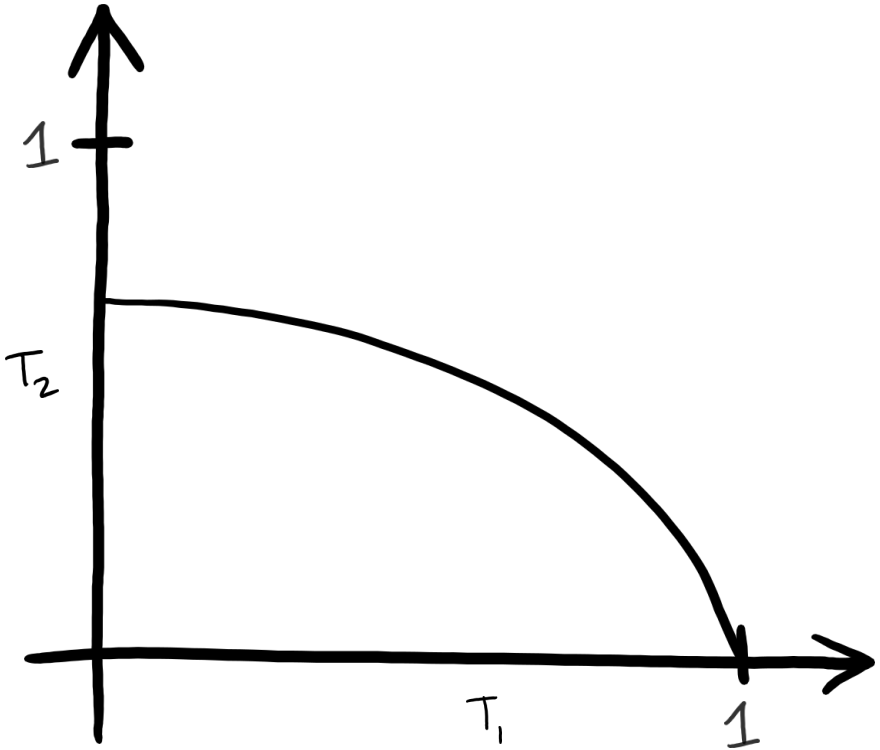

Want: Pareto frontier

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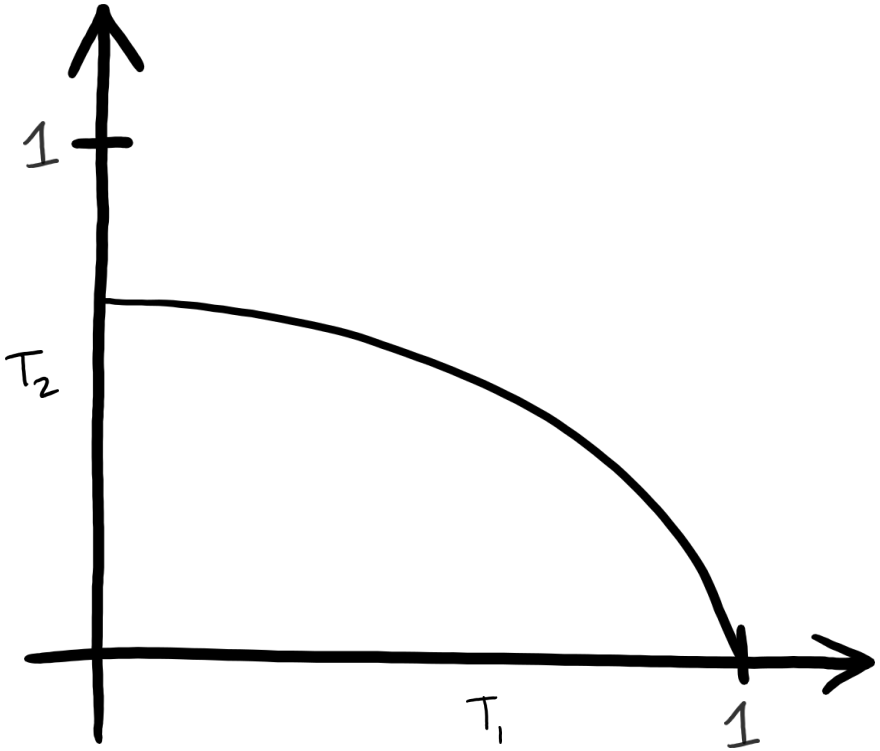

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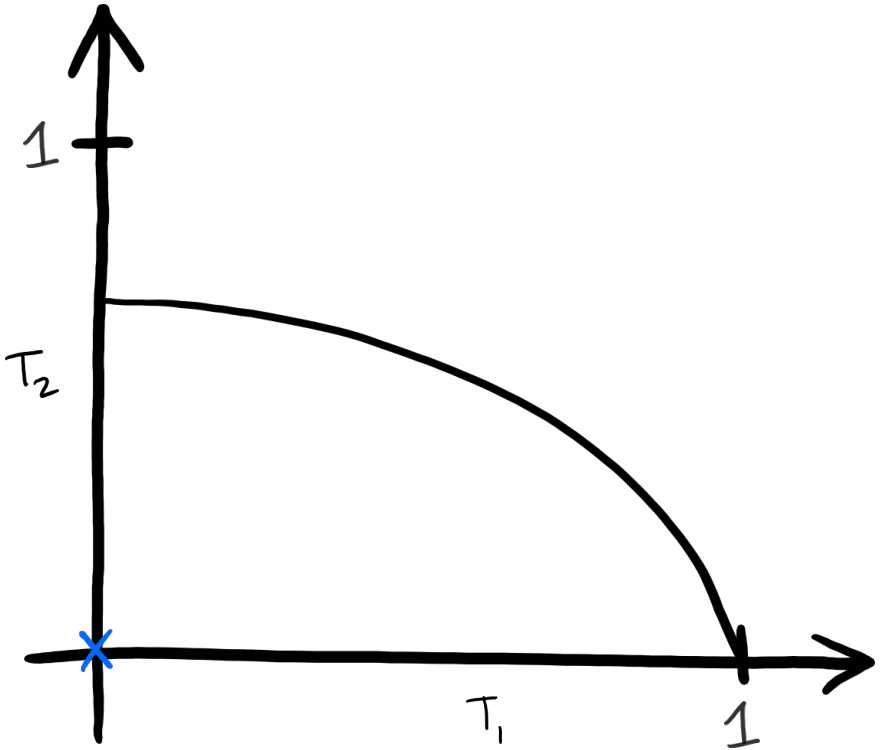

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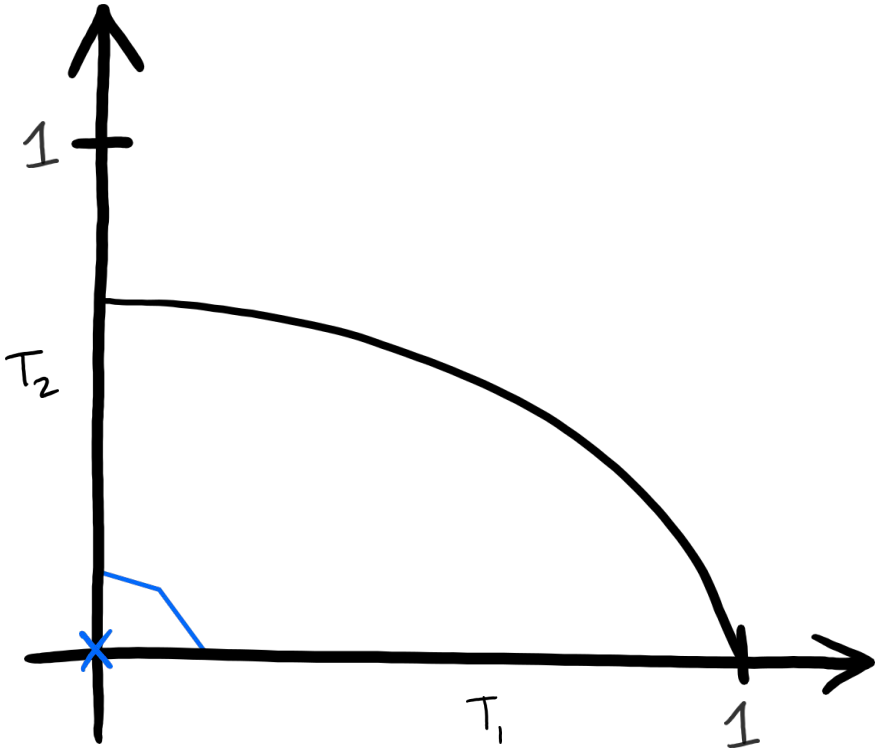

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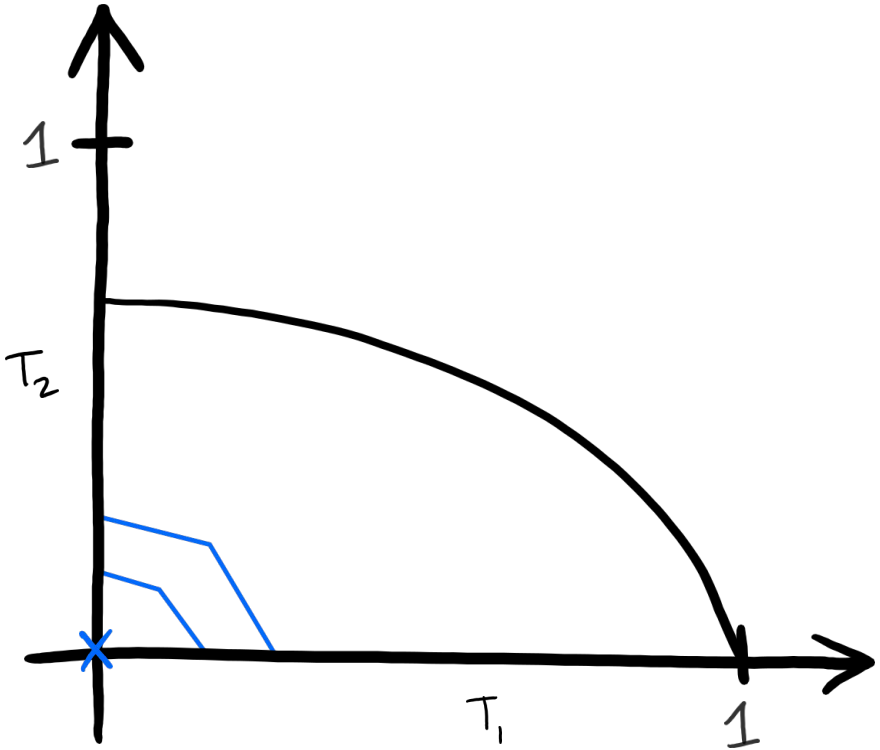

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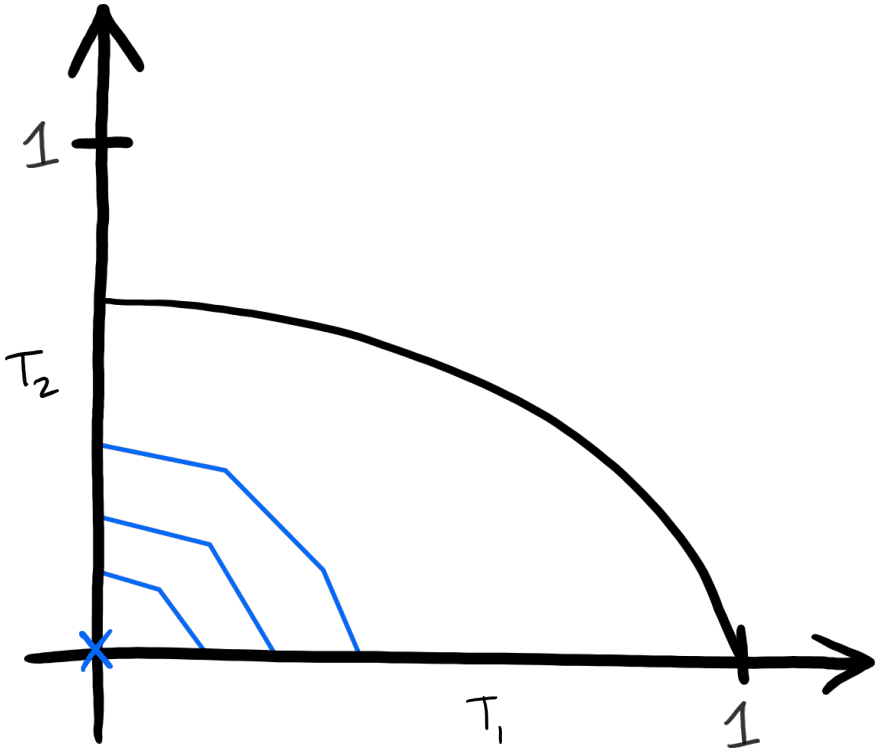

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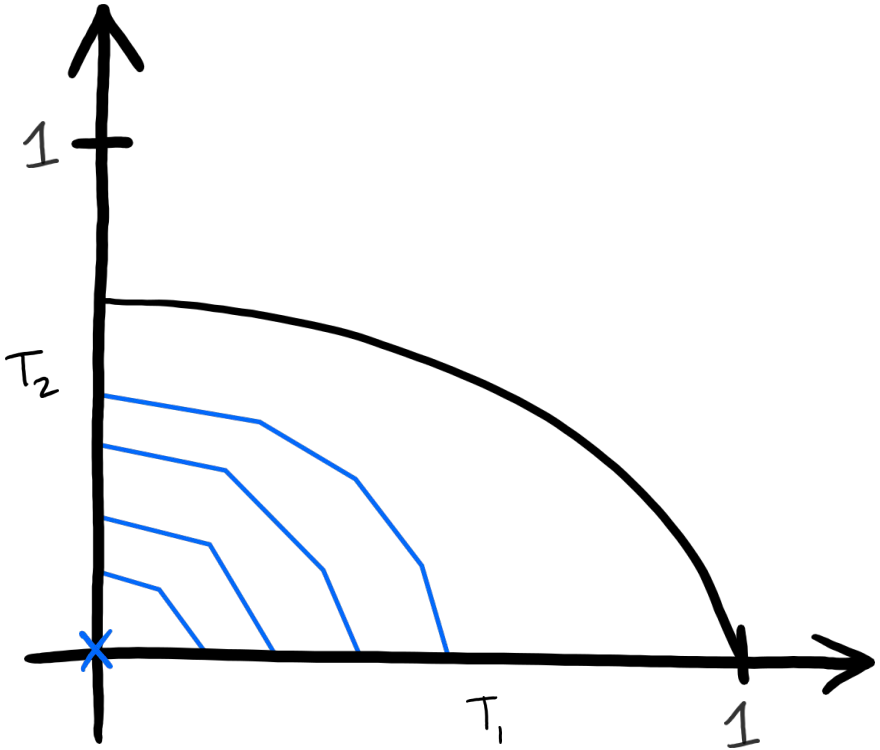

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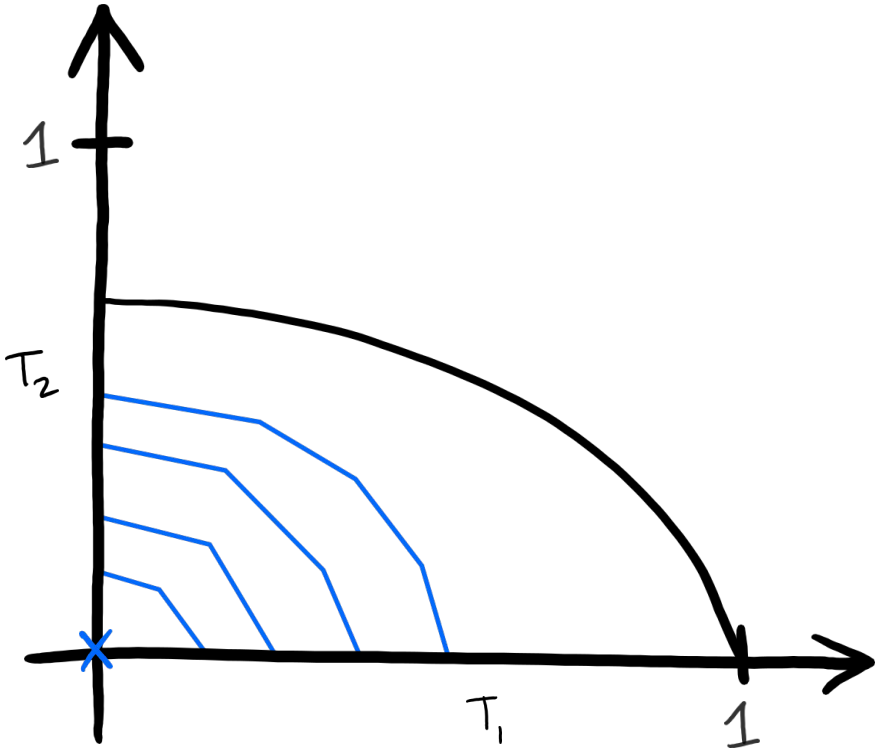

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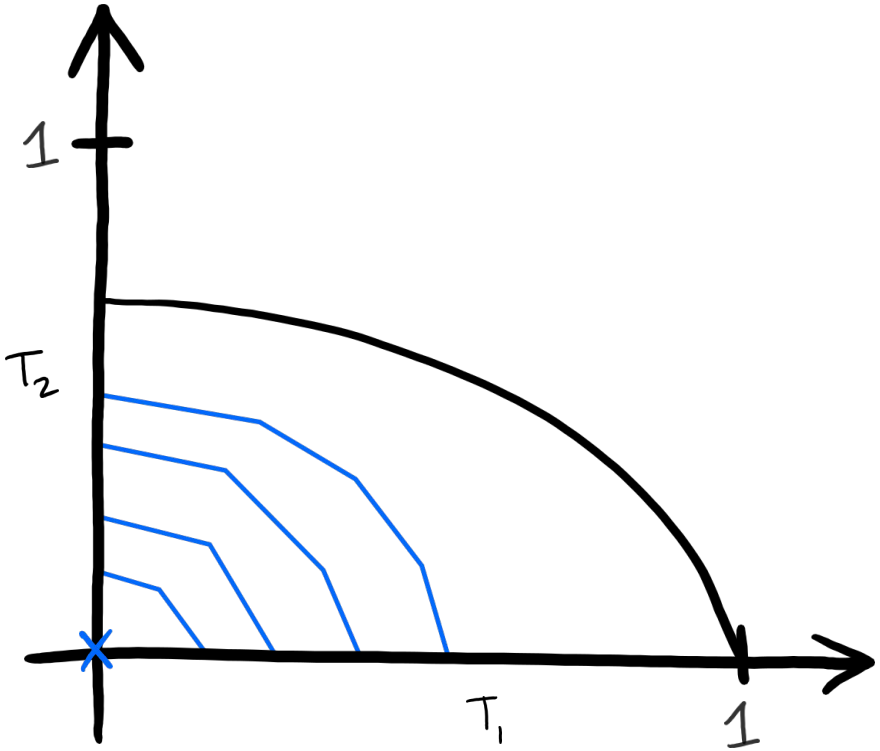

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Problem: When to stop?
Solution: Convergent over-approximation

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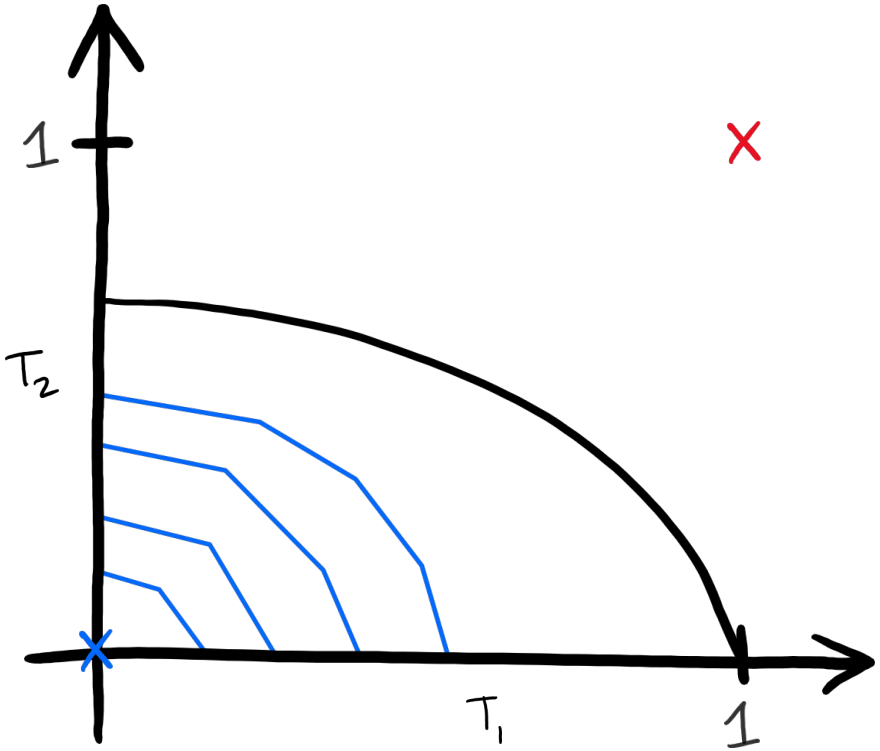

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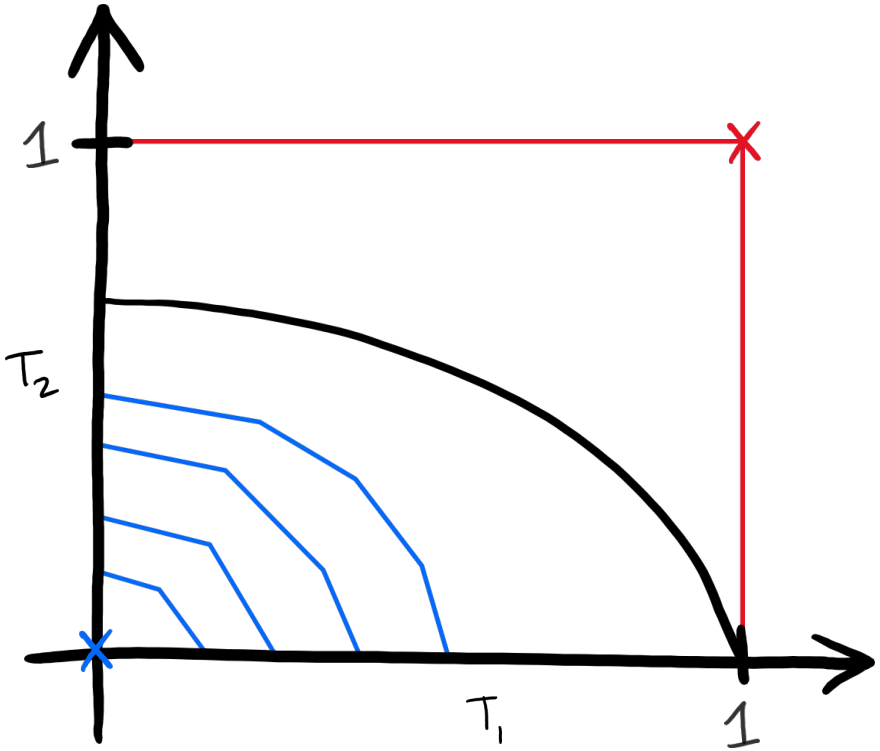

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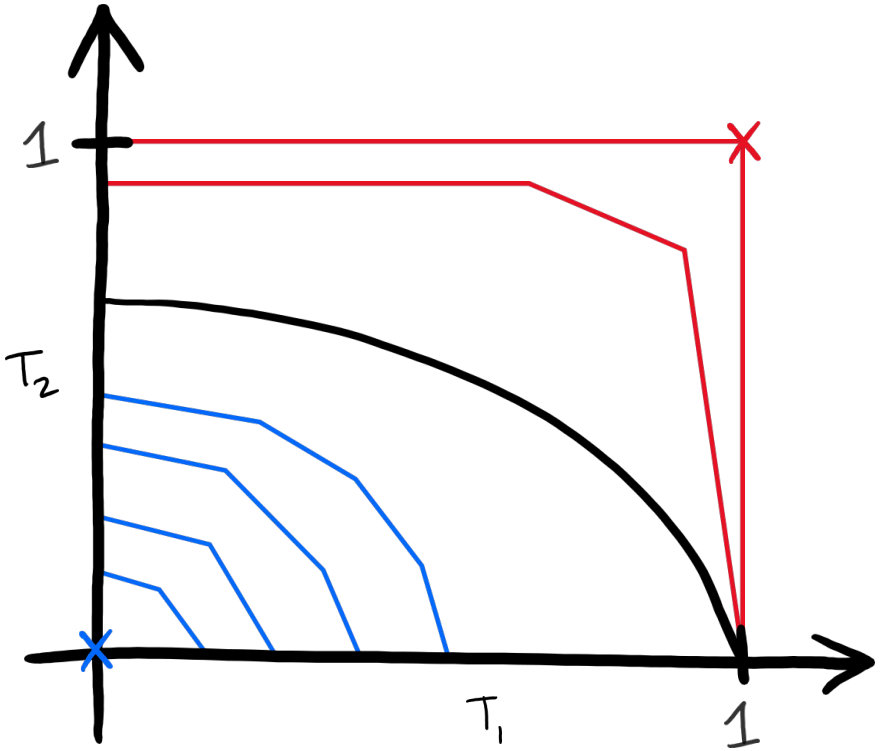

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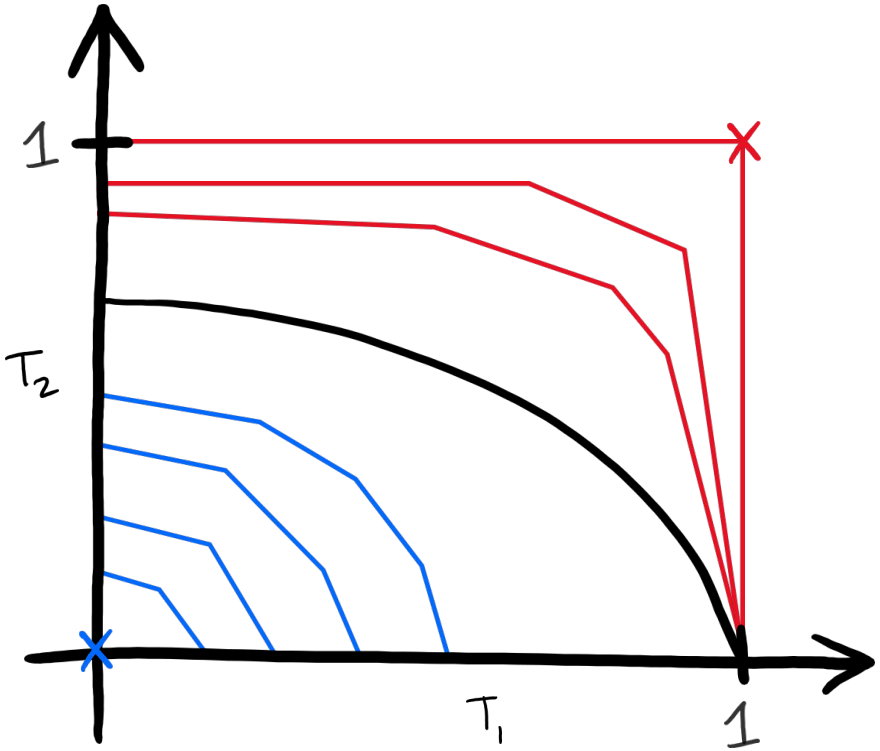

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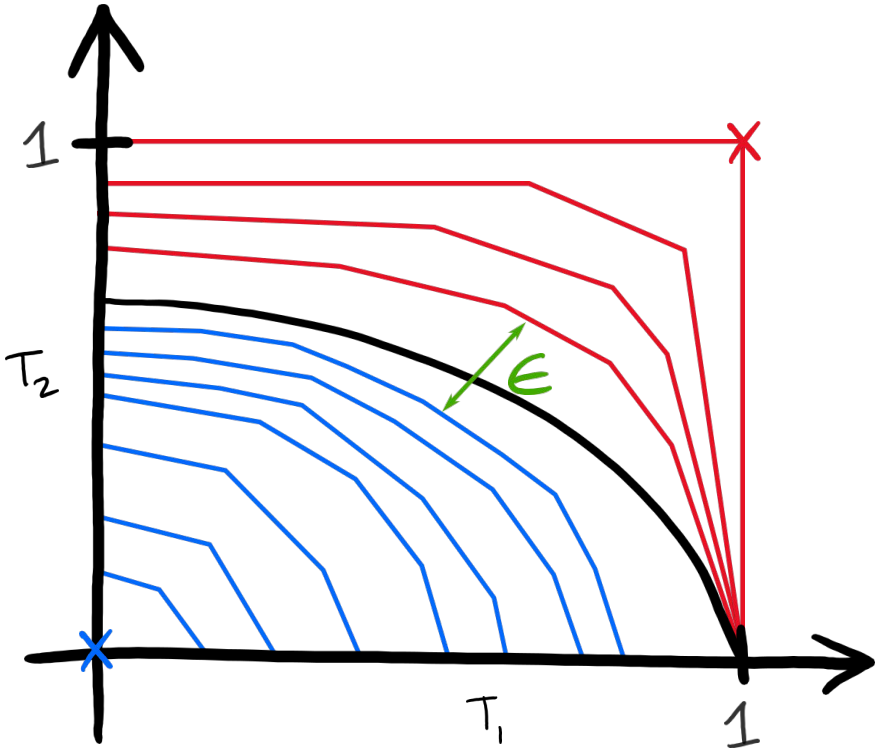

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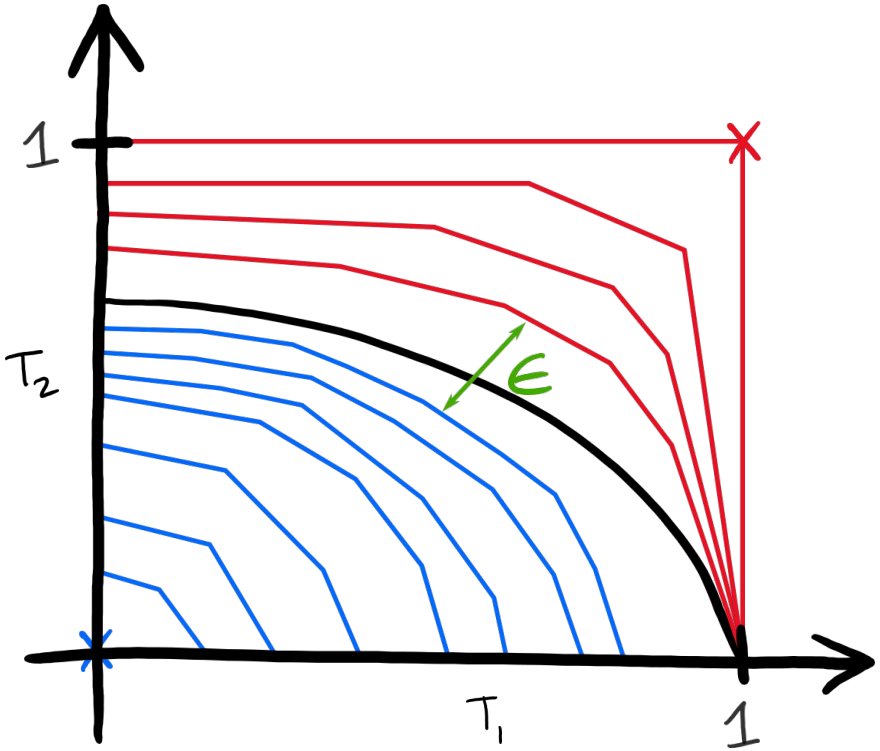

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Approximate values of generalized-reachability stochastic games for arbitrarily small precision.

Things to understand

- Stochastic game, value iteration

https://www.poornima.edu.in/wp-content/uploads/2020/08/greentick.jpg

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- Problems when computing over-approximation (greater fixpoints) 
- Single-dimensional solution

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- Single-dimensional solution  [+Paper]

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- Extension to multiple dimensions

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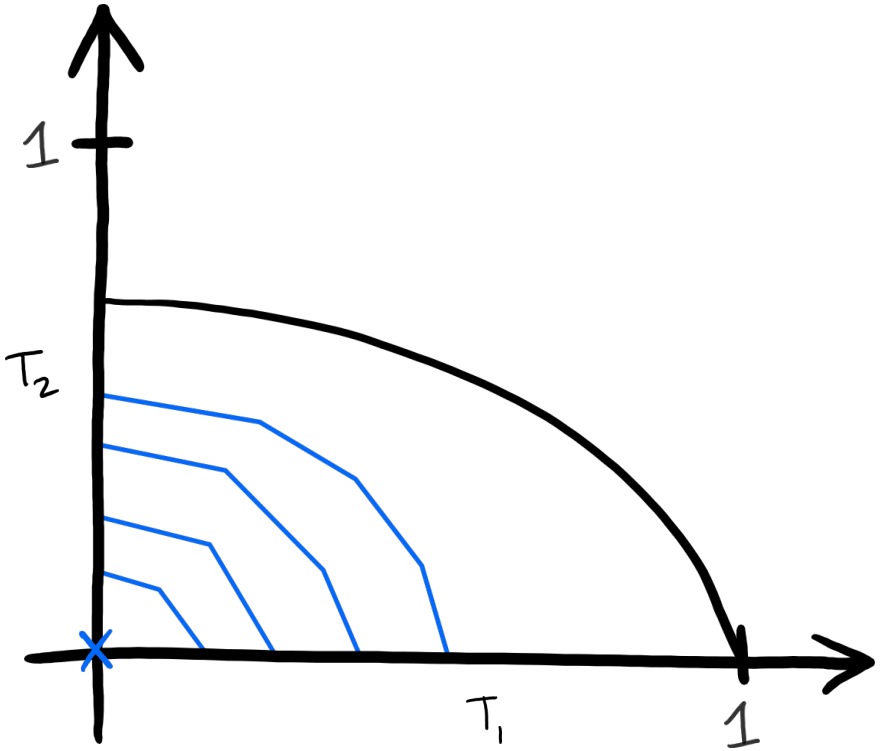
Implementation



We have:

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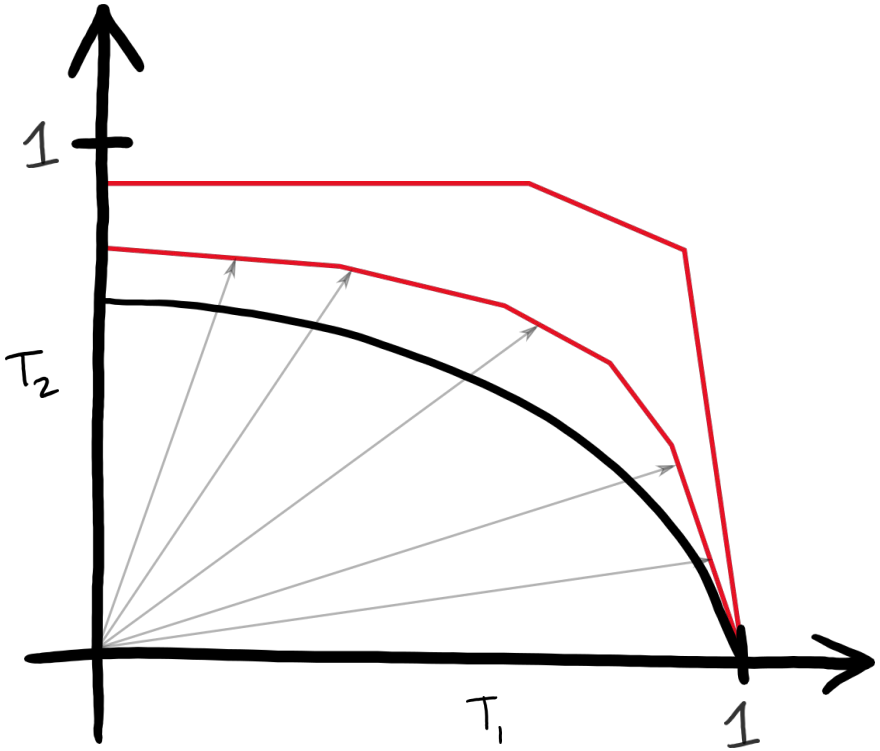

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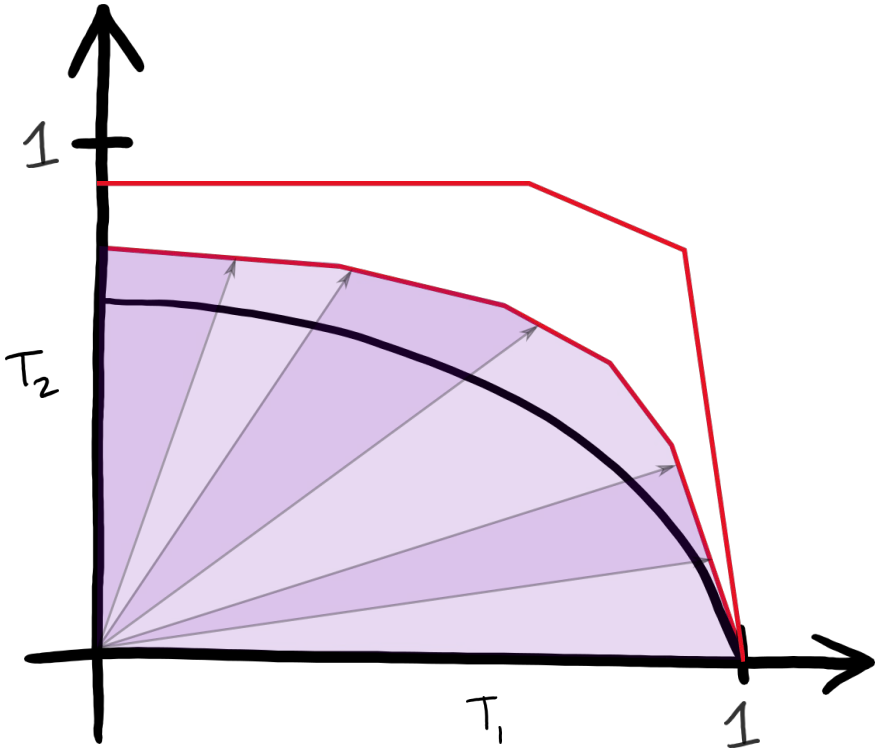

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More information

Paper available at:
https://dl.acm.org/doi/10.1145/3373718.3394761

Video presentation of the results:
https://www.youtube.com/watch?v=my7tOrom1Fg