

Optimal Budget Aggregation with Single-Peaked Preferences

Felix Brandt ¹ **Matthias Greger** ¹
Erel Segal-Halevi ² Warut Suksompong ³

¹Technical University of Munich

²Ariel University

³National University of Singapore

The 25th ACM Conference on Economics and Computation
July 9, 2024

Budget Aggregation with Single-Peaked Preferences



divisible budget



m public projects (*no caps*)



peak

n citizens

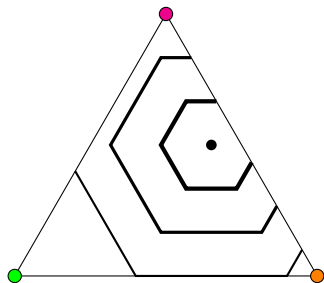
Preference Model(s)

- Each agent reports her (unique) most preferred distribution $\mathbf{p}_i = (p_{i,j})$.
- A mechanism returns a unique distribution \mathbf{q} of the budget given the agents' peaks.
- Agents have *star-shaped* (Border and Jordan, 1983) utility functions, i.e., for any distribution $\mathbf{q} \neq \mathbf{p}_i$ and $\lambda \in (0, 1)$,

$$u_i(\mathbf{p}_i) > u_i(\lambda\mathbf{p}_i + (1 - \lambda)\mathbf{q}) > u_i(\mathbf{q}).$$

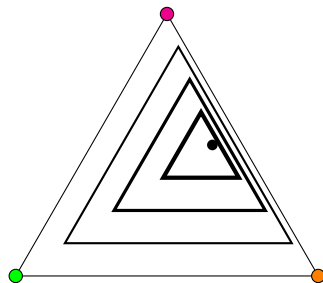
- ℓ_1 preferences: $u_i(\mathbf{q}) = -\sum_j |p_{i,j} - q_j|$.
- Leontief preferences: $u_i(\mathbf{q}) = \min_j \frac{q_j}{p_{i,j}}$.

Comparison of ℓ_1 and Leontief preferences



ℓ_1 preferences

“uniform” representation
of all projects



Leontief preferences

better representation
of “small” projects



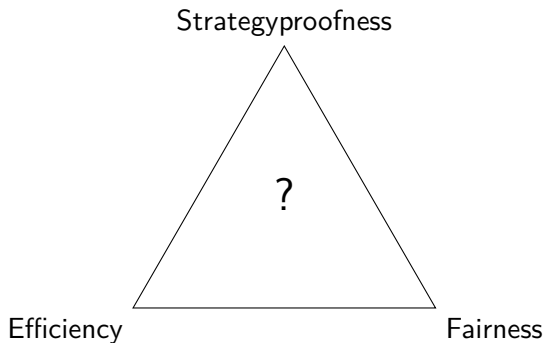
Preferences over distributions

- Probabilistic social choice (e.g., Gibbard, 1977) and fair mixing (e.g., Bogomolnaia et al., 2005) consider degenerate peak(s).
- Idea of having non-degenerate peaks can be traced back to Intriligator (1973).
- Lindner et al. (2008) and Goel et al. (2019) introduced ℓ_1 preferences.
- Brandt et al. (2023) first considered Leontief preferences.

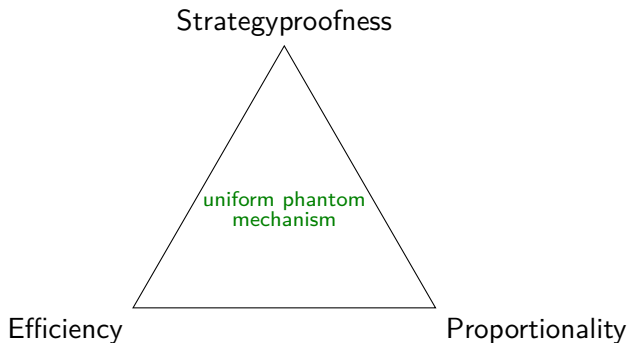
Known mechanisms

- For $m = 2$, characterization of generalized median rules for single-peaked preferences by Moulin (1980).
- Freeman et al. (2021) introduced the *independent markets mechanism* for ℓ_1 preferences.
- Lindner et al. (2008) and Goel et al. (2019) investigated utilitarian welfare maximization for ℓ_1 preferences.

Optimal Budget Aggregation



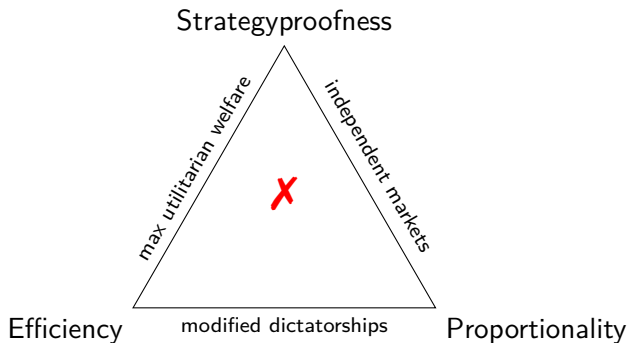
Here, we consider *proportionality* as a fairness axiom: If all agents have degenerate peaks (i.e., $p_{i,j} \in \{0, 1\}$ for all i, j), then $q_j = \sum_i p_{i,j}/n$.



Theorem

For $m = 2$, the only continuous mechanism that satisfies strategyproofness and proportionality is the uniform phantom mechanism (independent of the underlying utility model).

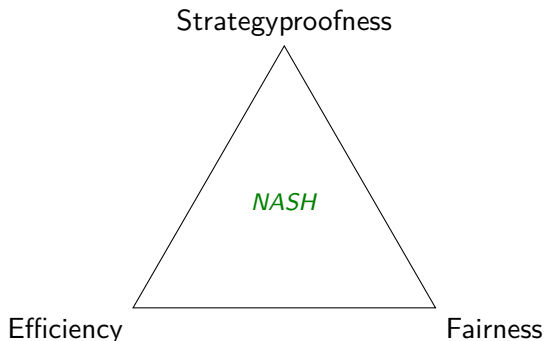
More than Two Projects - ℓ_1 preferences



Theorem

With ℓ_1 preferences, no mechanism satisfies efficiency, strategyproofness, and proportionality when $m \geq 3$ and $n \geq 3$.

More than Two Projects - Leontief preferences



Theorem

With Leontief preferences, maximizing the product of utilities (NASH) is the only continuous mechanism that satisfies group-strategyproofness and a core-based fairness notion stronger than proportionality.

Summary and Future Directions

- For $m = 2$, the uniform phantom mechanism seems like the undisputed winner.
- For $m > 2$, the class of star-shaped preferences is rich enough to contain both “impossibilities” and “possibilities”.
 - Impossibilities for ℓ_1 (and also ℓ_∞) preferences.
 - Characterization of *NASH* for Leontief preferences.
- Investigate other utility models from the class of star-shaped preferences.
- Consider other fairness axioms.
- Work towards a general understanding of star-shaped preferences.

- A. Bogomolnaia, H. Moulin, and R. Stong. Collective choice under dichotomous preferences. *Journal of Economic Theory*, 122(2):165–184, 2005.
- K. C. Border and J. S. Jordan. Straightforward elections, unanimity and phantom voters. *Review of Economic Studies*, 50(1):153–170, 1983.
- F. Brandt, M. Greger, E. Segal-Halevi, and W. Suksompong. Balanced donor coordination. In *Proceedings of the 24th ACM Conference on Economics and Computation (ACM-EC)*, page 299, 2023.
- R. Freeman, D. M. Pennock, D. Peters, and J. W. Vaughan. Truthful aggregation of budget proposals. *Journal of Economic Theory*, 193:105234, 2021.
- A. Gibbard. Manipulation of schemes that mix voting with chance. *Econometrica*, 45(3):665–681, 1977.
- A. Goel, A. K. Krishnaswamy, S. Sakshuwong, and T. Aitamurto. Knapsack voting for participatory budgeting. *ACM Transactions on Economics and Computation*, 7(2):8:1–8:27, 2019.
- M. D. Intriligator. A probabilistic model of social choice. *Review of Economic Studies*, 40(4):553–560, 1973.
- T. Lindner, K. Nehring, and C. Puppe. Allocating public goods via the midpoint rule. In *Proceedings of the 9th International Meeting of the Society of Social Choice and Welfare*, 2008.
- H. Moulin. On strategy-proofness and single peakedness. *Public Choice*, 35(4):437–455, 1980.